Math 115A - Spring 2019

Practice Exam 1

Full Name: ______________________________

UID: ______________________________

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- All work including proofs should be well organized and clearly written using complete sentences.
- You may use the provided scratch paper, however this work will not be graded unless very clearly indicated there and in the exam.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	10	
2	15	
3	10	
4	10	
Total:	45	

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated here and in the exam.

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1. (10 points) True or False: Prove or disprove the following statements.

   (a) If U_1, U_2, and W are subspaces of a finite-dimensional vector space V such that $U_1 + W = U_2 + W$, then $U_1 = U_2$.

   (b) Fix an $n \times n$ matrix B and let $W = \{A \in M_{n\times n}(\mathbb{F}) \mid AB = BA\}$. Then W is a subspace of $M_{n\times n}(\mathbb{F})$.

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2. (15 points) True or False: Prove or disprove the following statements.

(a) The set $W = \{(a, b, c) \in \mathbb{R}^3 \mid a^2 + b^2 + c^2 = 0\}$ is a subspace of $\mathbb{R}^3$.

(b) The set $W = \{(a, b, c) \in \mathbb{R}^3 \mid a + b + c = 0\}$ is a subspace of $\mathbb{R}^3$.

(c) There exists a linear transformation $T : \mathbb{F}^5 \to \mathbb{F}^2$ with

$$\ker T = \{(a, b, c, d, e) \in \mathbb{F}^5 \mid a = b \text{ and } c = d = e\}.$$

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3. (10 points) True or False: Prove or disprove the following statements.

(a) Let $S = \{(1, -1, 0), (0, 1, -1), (1, 1, 1)\} \subseteq \mathbb{R}^3$. The list S is a basis for $\mathbb{R}^3$.

(b) Let $B = \{(1, -1, 0), (0, 1, -1), (1, 1, 1)\} \subseteq (\mathbb{F}_2)^3$. The list B is a basis for $(\mathbb{F}_2)^3$.

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4. (10 points) True or False: Let W_1 and W_2 be subspaces of a vector space V over a field $\mathbb{F}$. Prove or disprove the following sets are subspaces of V.

(a) The intersection of W_1 and W_2, given by

$$W_1 \cap W_2 = \{v \in V \mid v \in W_1 \text{ and } v \in W_2\}.$$

(b) The difference of W_1 from W_2, given by

$$W_2 - W_1 = \{v \in V \mid v \in W_2 \text{ and } v \notin W_1\}.$$

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