Math 115A - Spring 2019
Practice Exam 2

Full Name: ______________________________

UID: ______________________________

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- All work including proofs should be well organized and clearly written using complete sentences.
- You may use the provided scratch paper, however this work will not be graded unless very clearly indicated there and in the exam.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	10	
2	10	
3	15	
4	15	
Total:	50	

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated here and in the exam.

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1. (10 points) True or False: Prove or disprove the following statements.

   (a) If $T : V \to W$ is a linear map between two n-dimensional vector spaces then T is onto if and only if T is one-to-one.

   (b) If $T : V \to W$ is a linear map between two finite-dimensional vector spaces then T is an isomorphism if and only if T maps any basis β for V to a basis $T(\beta)$ for W.

/10

2. (10 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the projection onto the x-axis along the line $y = 2x$.

   (a) Give a basis for $\mathbb{R}^2$ consisting of eigenvectors for T and find their corresponding eigenvalues.

   (b) Find the matrix T in the standard basis for $\mathbb{R}^2$.

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3. (15 points) Let $\beta = \{1, x, x^2\}$ and $\beta' = \{1 + x + x^2, x + x^2, x^2\}$ be bases of $P_2(\mathbb{R})$.

(a) Find the change of coordinate matrix from β' to β.

(b) Find the characteristic polynomial for the matrix found in part (a).

(c) Find the change of coordinate matrix from β to β'.

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4. (15 points) Let $V = P_3(\mathbb{R})$ and $W = M_{2\times 2}(\mathbb{R})$. Let

$$\beta = \{1, x, x^2, x^3\}$$
$$\gamma = \left\{ w_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, w_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, w_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, w_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

be the standard bases. Consider the linear map $T : V \to W$ defined by

$$T(ax^3 + bx^2 + cx + d) = \begin{pmatrix} a+b & c+d \\ a+c & b+c \end{pmatrix}.$$

(a) Determine $M = [T]_\beta^\gamma$.

(b) Prove that T is an isomorphism.

(c) Prove that V and W are isomorphic without using T.

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