All problems are to be written up clearly and thoroughly, using complete sentences. This assignment is due in discussion at 2pm on Thursday, May 2nd.

For all T/F problems on the homework, provide a brief justification for your answer. That may be citing an appropriate theorem or providing a counterexample.

1. Section 2.2 problems 1, 2 a, c, f, 3, 4, 10, 11, 12, 14, 15, 16

2. Let V and W be vector spaces over $\mathbb{F}$. Define the set
$$V \times W = \{(v, w) \mid v \in V \text{ and } w \in W\}.$$
This is called the *product* of V and W.

   (a) Show that $V \times W$ is a vector space.

   (b) Define a map $\iota_V : V \to V \times W$ by $\iota_V(x) = (x, 0)$. Show that ι_V is an injective linear map. Note that we can define a similar map ι_W.

   (c) If $U \subseteq V$ is a subspace, show that $U \times W$ is a subspace of $V \times W$.

   (d) Show that $V \times W = (V \times \{0\}) \oplus (\{0\} \times W)$. Notice we can consider $V \times \{0\}$ to be a copy of V in $V \times W$. For this reason, mathematicians often write $V \oplus W$ for $V \times W$ and refer to it as the *external direct product* of V and W.

3. Let V and W be vector spaces over $\mathbb{F}$. Define $\mathrm{Hom}(V, W)$ to be the set of linear maps from V to W. Show that $\mathrm{Hom}(V, W)$ is itself a vector space over $\mathbb{F}$.

4. As a special case of the definition above, if we take $W = \mathbb{F}$ then we write $V^* = \mathrm{Hom}(V, \mathbb{F})$ and call it the *dual vector space* to V. If V is finite-dimensional and B is a basis for V, construct a basis for $V^* = \mathrm{Hom}(V, \mathbb{F})$.

5. Let $T : V \to W$ be an injective linear map. Show the following: if we consider T instead as a linear map $V \to \operatorname{im} T$ (just restrict the codomain), then it defines an isomorphism. This shows that under these circumstances $V \cong \operatorname{im} T$.