

All problems are to be written up clearly and thoroughly, using complete sentences. This assignment is due in discussion at 2pm on Thursday, May 9th.

For all T/F problems on the homework, provide a brief justification for your answer. That may be citing an appropriate theorem or providing a counterexample.

1. Section 2.3 problems 1, 3, 4, 9, 11, 12, 16, 17
2. Section 2.4 problems 1, 2a, c, e, 3, 7, 14, 15, 17, 24
3. Let V be a finite-dimensional vector space and let W be a subspace. Show that V and $W \times V/W$ are isomorphic by constructing an explicit isomorphism (rather than simply computing the dimensions).

For HW 5 problem 3, I probably should have said that the product of vector spaces is just the underlying Cartesian product of sets with pointwise addition and scalar multiplication. If I take vector spaces over the same field U and V , then $U \times V$ has elements of the form (u, v) where u is in U and v is in V . We add coordinatewise so $(u, v) + (u', v') = (u+u', v+v')$ and scalar multiply $a(u, v) = (au, av)$. For number 3 in the homework, we read $W \times V/W$ as $W \times (V/W)$ so elements of $W \times V/W$ are of the form $(w, v+W)$, since elements of V/W are cosets of the form $v+W$.

4. Let V be a finite-dimensional vector space over \mathbb{F} and let $V^* = \text{Hom}(V, \mathbb{F})$ be its dual vector space. For W a subset of V , we define the **annihilator** of W to be the set

$$W^0 = \{f \in V^* \mid f(x) = 0 \text{ for all } x \in W\}.$$

(a) Show that W^0 is a subspace of V^* .

(b) For subspaces $W_1, W_2 \subseteq V$, show that $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$.