All problems are to be written up clearly and thoroughly, using complete sentences. This assignment is due in discussion at 2pm on Thursday, May 16th.

For all T/F problems on the homework, provide a brief justification for your answer. That may be citing an appropriate theorem or providing a counterexample.

1. Section 2.5 problems 1, 2, 3 a, c, e, 4, 5, 6, 7, 10

2. Section 4.4 problems 1, 5, 6

3. Section 5.1 problems 1, 2 a, b, c, 3

4. A differential operator on $\mathbb{R}_n[x]$ (the vector space of polynomials with degree less than or equal to n) is a linear combination of expressions of the form $x^a \frac{d^b}{dx^b}$ where $a - b \leq 0$ (otherwise the degree would potentially increase). We can consider a differential operator as a linear operator $\mathbb{R}_n[x] \to \mathbb{R}_n[x]$.

   (a) Let $D : \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ be the differential operator given by $2 - 4\frac{d}{dx} + 2x^2\frac{d^2}{dx^2}$. Find the matrix of D relative to the basis $\{x^2, (x-1)^2, (x+1)^2\}$.

   (b) Suppose $E : \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ is a differential operator and that the matrix of E relative to the basis $\{1, x, x^2\}$ is the following matrix.

   $$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

   What is E as a differential operator?