

All problems are to be written up clearly and thoroughly, using complete sentences. This assignment is due in discussion at 2pm on Thursday, May 30th.

For all T/F problems on the homework, provide a brief justification for your answer. That may be citing an appropriate theorem or providing a counterexample.

1. Section 5.2 problems 7, 8, 9, 10, 11, 12, 13, 18, 19, 20
2. Section 6.1 problems 1, 2, 3, 4, 8, 9, 10, 12, 16, 17, 18, 23, 29

For a collection U_i for $1 \leq i \leq k$ of subspaces of a vector space V , we call V the *direct sum* of the subspaces and write

$$V = U_1 \oplus U_2 \oplus \cdots \oplus U_k$$

if $U_i \cap \sum_{i \neq j} U_j = \{0\}$ and $V = U_1 + U_2 + \cdots + U_k$. This second condition means that every vector $v \in V$ can be written as a sum $v = \sum_{i=1}^k u_i$ for some $u_i \in U_i$.

3. Suppose V is a finite-dimensional vector space and $T : V \rightarrow V$ is diagonalizable. If T has eigenvalues $\lambda_1, \dots, \lambda_k$ show that V decomposes as the direct sum of its eigenspaces, i.e. show that

$$V = E_{\lambda_1} \oplus E_{\lambda_2} \oplus \cdots \oplus E_{\lambda_k}.$$