Solve the following problems. This assignment will not be collected.

1. From the book:

   Section 7.3 problems 8, 9, 12, 14

   Section 6.8 problems 5 a, b

2. Show that every matrix $A \in M_{n\times n}(\mathbb{R})$ can be written as the sum of a symmetric matrix and a skew-symmetric matrix (i.e. $M^t = -M$) in a unique way.

3. Let $\langle\ ,\ \rangle$ be a bilinear form on finite-dimensional vector space over $\mathbb{F} = \mathbb{R}$. Show there is a symmetric bilinear form $(\ ,\ )$ and a skew-symmetric bilinear form $[\ ,\ ]$ so that $\langle\ ,\ \rangle = (\ ,\ ) + [\ ,\ ]$.

4. Given a finite-dimensional vector space V over a field $\mathbb{F}$ and its dual $V^* = \mathrm{Hom}(V, \mathbb{F})$, consider the function
$$ev\colon V^* \times V \to \mathbb{F}$$
given by $(f, v) \mapsto f(v)$. Show that ev is a bilinear map and thus induces a linear map
$$\overline{ev}\colon V^* \otimes V \to \mathbb{F}.$$
Furthermore, suppose $\beta = \{v_1, \ldots, v_n\}$ is a basis for V and $\beta^* = \{w_1, \ldots, w_n\}$ its dual basis. Find the values of $\overline{ev}(w_i \otimes v_j)$ for all $1 \leq i, j \leq n$.