

All problems are to be written up clearly and thoroughly, using complete sentences. This assignment is due in discussion at 2pm on Tuesday, February 4th.

For all T/F problems on the homework, provide a brief justification for your answer. That may be citing an appropriate theorem or providing a counterexample.

1. From the book:

Section 6.3 problems 9, 12.

Section 6.4 problems 1, 7, 14.

Section 6.5 problems 1, 5.

An *isometry* is a linear operator  $T : V \rightarrow V$  on an inner product space such that  $\langle T(x), T(y) \rangle = \langle x, y \rangle$  for all pairs  $x, y \in V$ .

2. Let  $V$  be a finite-dimensional inner product space over  $\mathbb{F} = \mathbb{R}$  or  $\mathbb{C}$ .

(a) Fix  $y \in V$  and suppose  $\langle x, y \rangle = 0$  for all  $x \in V$ . Show that  $y = 0$ .

(b) Let  $T : V \rightarrow V$  be an isometry. Prove that  $T$  is an isomorphism.

(c) Find all isometries  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  that have  $\det T = 1$ .