All problems are to be written up clearly and thoroughly, using complete sentences. This assignment is due in discussion at 2pm on Tuesday, March 10th.

For all T/F problems on the homework, provide a brief justification for your answer. That may be citing an appropriate theorem or providing a counterexample.

1. From the book:

   Section 7.3 problems 1, 2 a, b, d, 3 b, 5

2. Let $T\colon V \to V$ be a linear operator on an n-dimensional vector space V. Suppose that $\dim(\ker(T^{n-1})) \neq \dim(\ker(T^n))$. Show that $\dim(\ker(T^k)) = k$ for every $0 \leq k \leq n$.

3. Let $A = \begin{pmatrix} 2 & -1 & 5 \\ 0 & 0 & -9 \\ 0 & 1 & 6 \end{pmatrix}$.

   (a) Find the eigenvalues of A.

   (b) Find the dimensions of the generalized eigenspaces of A.

   (c) Find Jordan canonical bases for the generalized eigenspaces.

   (d) Put these bases together to give a Jordan canonical basis of $\mathbb{R}^3$ for A and write A in Jordan canonical form.

4. Let $J_m(\lambda)$ and $J_m(\mu)$ be $m \times m$ Jordan blocks corresponding to eigenvalues λ and μ. Show that
$$J_m(\lambda)J_m(\mu) = J_m(\mu)J_m(\lambda).$$

5. Let $A = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$. Find $\lim\limits_{n\to\infty} A^n$.