# Math 115B - Winter 2020
# Practice Midterm Exam

**Full Name:** ______________________________

**UID:** ______________________________

**Instructions:**

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- All work including proofs should be well organized and clearly written using complete sentences.
- You may use the provided scratch paper, however this work will not be graded unless very clearly indicated there and in the exam.
- Calculators are not allowed but you may have a $3 \times 5$ inch notecard.

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| Page | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| Total: | 40 | |

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated here and in the exam.

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1. (10 points) True or False: Prove or disprove the following statements.

   Let $V$ be a finite-dimensional inner product space over $\mathbb{F} = \mathbb{C}$. Let $T : V \to V$ be a a linear operator and $T^*$ its adjoint.

   (a) The linear operator $S = T + T^*$ is diagonalizable.

   (b) If $T$ is normal then $||Tv|| = ||T^*v||$ for all $v \in V$.



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2. (10 points) Let $V$ be a finite-dimensional vector space and let $T$ and $S$ be linear operators on $V$. Suppose $V$ is a $T$-cyclic subspace of itself. Show that $T$ and $U$ commute if and only if $U = g(T)$ for some polynomial $g(t)$.



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3. (10 points) Let $T\colon V \to V$ be a linear operator on a finite-dimensional vector space over a field $\mathbb{F}$. Let $T^t\colon V^* \to V^*$ be its dual. Show that a subspace $W \subseteq V$ is $T$ invariant if and only if $W^0$ is $T^t$-invariant.



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4. (10 points) True or False: Prove or disprove the following statements.

(a) Let $V$ be a finite-dimensional inner product space and let $T\colon V \to V$ be a linear operator. If all the eigenvalues of $T$ are 1, then $T$ must be an isometry.

(b) Let $\beta = \{1, x, x^2\}$ be the standard basis for $V = P_2(\mathbb{R})$. There exists a basis for $V$ such that the dual basis for $V^*$ is given by $\{f_0, f_1, f_2\}$ with $f_0(p(x)) = p(0)$, $f_1(p(x)) = p(1)$, and $f_2(p(x)) = p(2)$.



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