Read Section 5.4 and answer the following. Your answers should be complete sentences but they may be brief. You are not expected to provide a complete proof unless the directions say something like "show" or "prove." Often, you may just cite a relevant theorem from the text.

1. Give an example of a linear operator $T\colon V \to V$ and subspace that is not T-invariant.

2. Show that for any linear operator $T\colon V \to V$ the eigenspace $E_\lambda = \ker(T - \lambda I)$ is a T-invariant subspace.

3. Suppose $p_T(x)$ is the characteristic polynomial of $T\colon V \to V$. What is the value of $p_T(T)$?

4. Let $T\colon V \to V$ be a linear operator and $V = W_1 \oplus W_2$ where $W_1, W_2 \subseteq V$ are both T-invariant subspaces. Suppose the characteristic polynomials for the restrictions T_{W_1} and T_{W_2} are $f(x)$ and $g(x)$. What is the characteristic polynomial for T?