

Read Section 6.6 and pages 53-54 on Lagrange interpolation, then answer the following.

1. Let  $f_i$  be the Lagrange polynomials in  $P_n(\mathbb{R})$  corresponding to some distinct scalars  $c_0, c_1, \dots, c_n \in \mathbb{R}$ . Let  $g \in P_n(\mathbb{R})$  be defined to be  $g = \sum_{i=0}^n b_i f_i$  for some (not necessarily distinct) scalars  $b_0, b_1, \dots, b_n$ . What is the value of the  $g(c_j)$ ?
2. Let  $T: V \rightarrow V$  be a linear operator on a finite-dimensional inner product space  $V$ .
  - (a) Show that if  $T$  is an orthogonal projection then  $\|T(v)\| \leq \|v\|$  for all  $v \in V$ . You may want to type something like `norm(v)` for  $\|v\|$ . (Hint: use the triangle inequality).
  - (b) Give an example of a projection for which this inequality does not hold.
  - (c) What can you say about  $T$  if the inequality is actually an equality for all  $v \in V$ ?