

Read Section 6.7 and page 374 where positive definite and positive semidefinite operators are defined, then answer the following questions.

Let V and W be finite-dimensional inner product spaces and let $T: V \rightarrow W$ be a linear map of rank r .

1. In your own words, what are the singular values of T ? How many singular values are there?
2. How do the singular values of T relate to the singular values of T^* ?
3. What is the pseudoinverse of T ?

Now let V be a finite-dimensional inner product space with $T: V \rightarrow V$ a self-adjoint linear operator.

4. Show that T is positive definite if and only if all of its eigenvalues are positive. Furthermore, show that T is positive semidefinite if and only if all of its eigenvalues are nonnegative.