

# Math 32A - Winter 2019

## Exam 2

Full Name: \_\_\_\_\_

UID: \_\_\_\_\_

Circle the name of your TA and the day of your discussion:

Qi Guo

Talon Stark

Tianqi (Tim) Wu

Tuesday

Thursday

**Instructions:**

- Read each problem carefully.
  - Show all work clearly and circle or box your final answer where appropriate.
  - Justify your answers. A correct final answer without valid reasoning will not receive credit.
  - Simplify your answers as much as possible.
  - Include units with your answer where applicable.
  - Calculators are not allowed but you may have a  $3 \times 5$  inch notecard.
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Page	Points	Score
1	32	
2	26	
3	22	
4	20	
Total:	100	

1. (20 points) Consider the surface defined by  $ze^{2x} + x^2y + y = 3 + 2e^{z+1}$ .

(a) Find an equation of the tangent plane to the surface at the point  $P = (0, 5, -1)$ .

(b) Find a vector equation for the line passing through the surface at  $P = (0, 5, -1)$  orthogonal to the plane found in part (a).

2. (12 points) Either give an example of a function  $f(x, y)$  with  $f_x(x, y) = 2x + y^2e^x$  and  $f_y(x, y) = x^2 + y^2e^x$  or show that no such function  $f$  can exist.

3. (16 points) Reparameterize the curve  $\mathbf{r}(t) = \langle \sqrt{15}t^2, \cos(t^2), \sin(t^2) \rangle$  where  $t \geq 0$  with respect to arc length.

4. (10 points) Show the following limit does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + 3y^4}$$

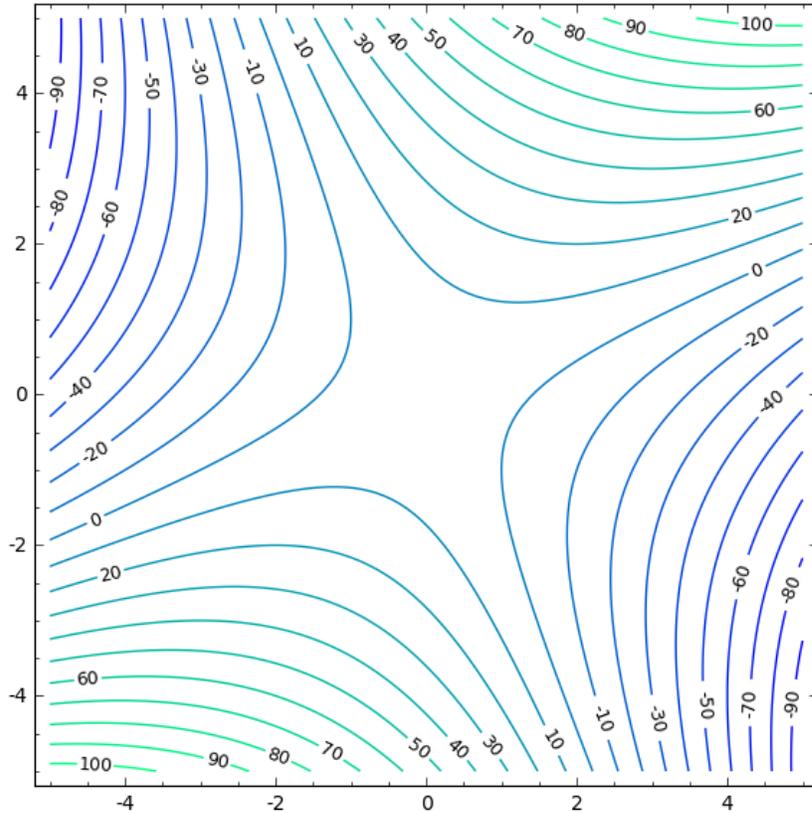
5. (22 points) Consider the function  $f(x, y) = \sqrt{10 - x^2 - 5y^2}$ .

(a) Use a linear approximation to  $f(x, y)$  at the point  $(2, 1)$  to estimate the value of  $f(1.95, 1.04)$ .

(b) Find the directional derivative of  $f$  at the point  $(2, 1)$  in the direction of  $\langle 4, 3 \rangle$ .

(c) Find the maximum rate of change of  $f$  at the point  $(2, 1)$ .

6. (20 points) Consider the contour plot for  $f(x, y)$  below.



(a) Determine the sign of each of the following derivatives.

$$f_x(3, 0) \text{ _____} \qquad f_y(3, 0) \text{ _____}$$

$$f_{xx}(3, 0) \text{ _____} \qquad f_{xy}(3, 0) \text{ _____} \qquad f_{yy}(3, 0) \text{ _____}$$

(b) Give the components of a unit vector in the direction of the steepest decline at the point  $(-1, 0)$ . (You may estimate as necessary.)

(c) Give the components of a unit vector orthogonal to  $\nabla f(2, 2)$ . (You may estimate as necessary.)

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated in the exam.