Math 32A - Winter 2019
Practice Exam 2

Full Name: ______________________________

UID: ______________________________

Circle the name of your TA and the day of your discussion:

Qi Guo   Talon Stark   Tianqi (Tim) Wu

Tuesday   Thursday

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. Suppose the elevation of a mountain above sea level at a point (x, y) is given by the function $z = 2000 - 2x^2 - 4y^2$ feet, where the positive x-axis points east, and the positive y-axis points north. A climber is at the point $P = (-20, 5, 1100)$.

   (a) (10 points) Suppose the climber uses a compass to walk northeast from P. At what rate will the climber initially ascend or descend?

   (b) (5 points) Suppose the climber wants to travel down the mountain from the point P as quickly as possible. In what direction should the climber set out from P? Find a unit vector in this direction.

   (c) (5 points) Find all possible directions the climber could walk to travel a level path from P (neither ascending nor descending). Give your answers as unit vectors.

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2. (10 points) Show that every plane that is tangent to the cone $z^2 = x^2+y^2$ passes through the origin.

3. (10 points) At a landscaping firm, a dirt pile starts as a cone with radius 10 meters and height 9 meters. As it is used up, the height decreases by 0.3 meters per day, but due to slippage the radius increases by 0.1 meters per day. What was the initial rate of dirt usage by the firm in cubic meters per day? *Hint:* You may use the fact that the volume of a cone is is $V = \frac{1}{3}\pi r^2 h$.

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4. (10 points) Show the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^6}{xy^3}$$

5. (10 points) Show the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + \sin^2 y}{2x^2 + y^2}$$

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6. Consider the curve $\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t \rangle$.

(a) (6 points) Find the length of the curve for $1 \le t \le 3$.

(b) (10 points) Find the Frenet frame for $\mathbf{r}(t)$ at the point $(1, \frac{2}{3}, 1)$.

(c) (4 points) Find the curvature of $\mathbf{r}(t)$ at the point $(1, \frac{2}{3}, 1)$.

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7. (20 points) Match each function with its graph on the next page and its contour plot on the following page.

   1. $f(x, y) = \sin(y)$ ________________

   2. $f(x, y) = (x^2 - y^2)^2$ ________________

   3. $f(x, y) = (x - y)^2$ ________________

   4. $f(x, y) = 3 - x^2 - y^2$ ________________

   5. $f(x, y) = \sin(4x)e^{-x^2-y^2}$ ________________

   6. $f(x, y) = \sin(x)\sin(y)e^{-x^2-y^2}$ ________________

   7. $f(x, y) = \dfrac{x}{1 + x^2 + y^2}$ ________________

   8. $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$ ________________

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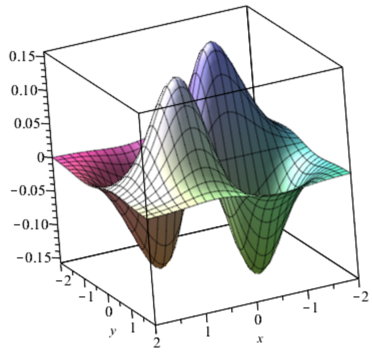
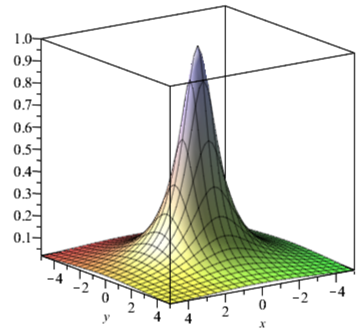

0.15
0.10
0.05
0
−0.05
−0.10
−0.15
−2 −1 0 1 2
y
2 1 0 −1 −2
x

(A)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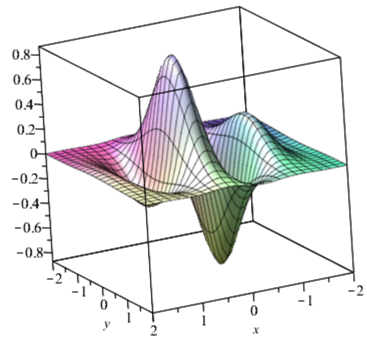
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
−4 −2 0 2 4
y
4 2 0 −2 −4
x

(B)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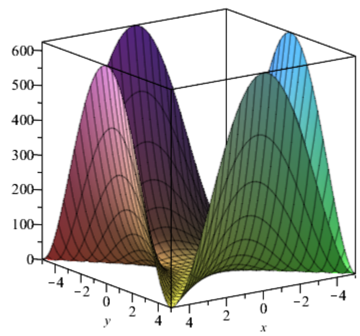
0.2
0
−0.2
−0.4
−0.6
−0.8
−2 −1 0 1 2
y
2 1 0 −1 −2
x

(C)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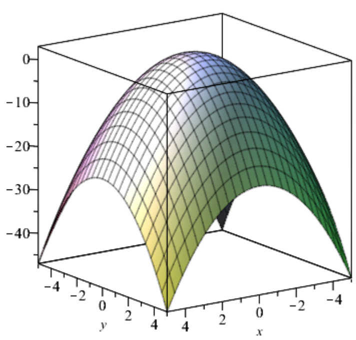
500
400
300
200
100
0
−4 −2 0 2 4
y
4 2 0 −2 −4
x

(D)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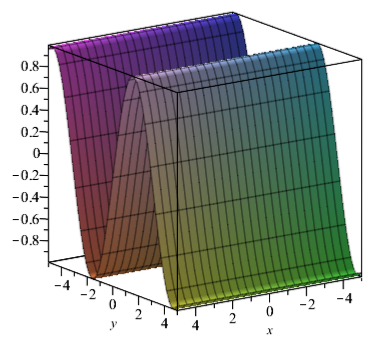
y
4 2 0 −2 −4
x

(E)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0.2
0
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−0.6
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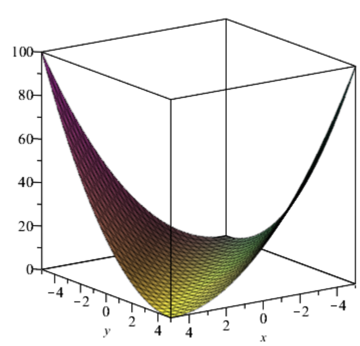
y
4 2 0 −2 −4
x

(F)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40
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0
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y
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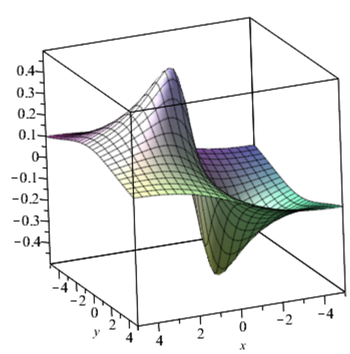
x

(G)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0.1
0
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−0.3
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−4 −2 0 2 4
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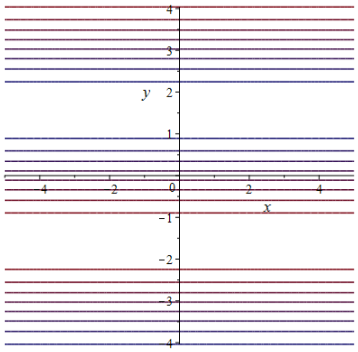
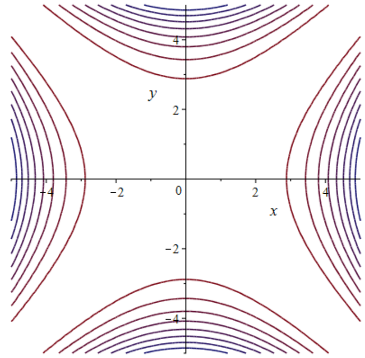
x

(H)

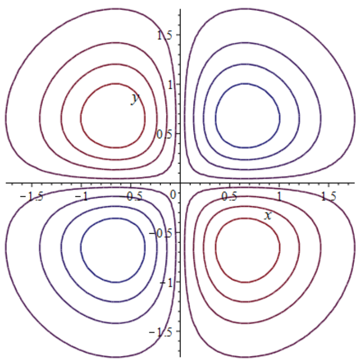
/0



(i)



(ii)



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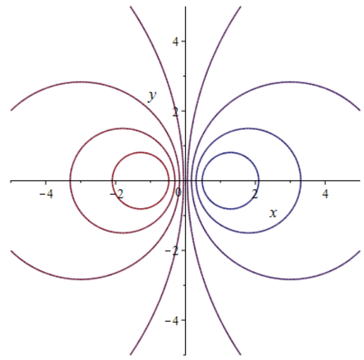

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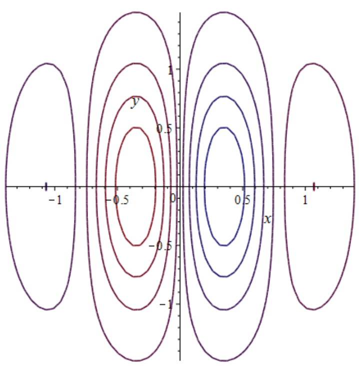

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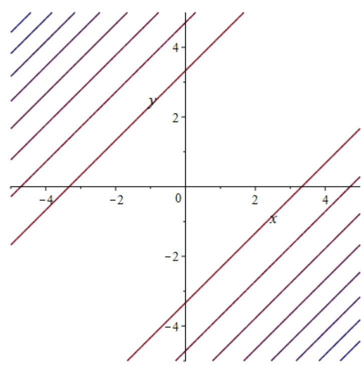

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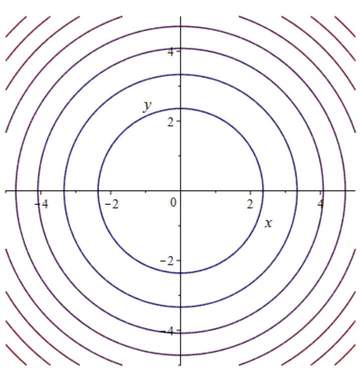

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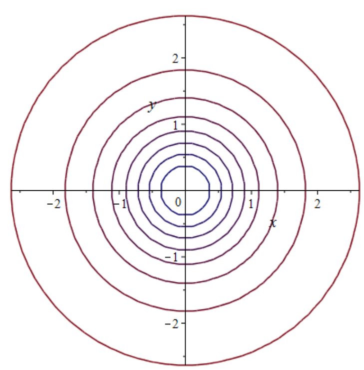

(viii)

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