Math 32A - Winter 2019
Practice Final Exam

Full Name: ______________________________

UID: ______________________________

Circle the name of your TA and the day of your discussion:

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Tuesday　　Thursday

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	10	
2	10	
3	8	
4	10	
5	12	
6	12	

Page	Points	Score
7	12	
8	10	
9	6	
10	10	
Bonus		
Total:	100	

1. (2 points) Suppose $\mathbf{u}$ is a unit vector and $\mathbf{v}$ is a vector with $||\mathbf{v}|| = 5$. If the angle θ between $\mathbf{u}$ and $\mathbf{v}$ has $\sin\theta = \frac{3}{5}$, find the length of $\mathbf{u} \times \mathbf{v}$.

2. (3 points) Given a curve with binormal $\mathbf{B}$, show that $\dfrac{d\mathbf{B}}{ds}$ is perpendicular to $\mathbf{B}$.

3. (5 points) Consider the planes $3x - 2y + z = 1$ and $2x + y - 3z = 3$, which intersect in a line L.

   (a) Notice that the point $P = (1, 1, 0)$ is in the intersection of the planes and so is on L. Use P to find a vector equation for L.

   (b) If θ is the angle between the planes, find $\cos\theta$.

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4. (5 points) Find the equation of the plane that passes through the point $(1, 2, 3)$ and contains the line given by the parametric equations $x = 3t, y = 1 + t, z = 2 - t$.

5. (2 points) Suppose that $w = f(x, y, z)$, $y = g(s, t)$, and $z = h(t)$. Write down the form of the chain rule you would use to compute $\partial w/\partial s$ and $\partial w/\partial t$.

6. (3 points) Find parametric equations for the line normal to the surface $\sin(xyz) = x + 2y + 3z$ at the point $(2, -1, 0)$.

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7. (3 points) For what values of x are the following vectors orthogonal?

$$\mathbf{v} = \langle x, x-1, x+1 \rangle \qquad \mathbf{w} = \langle 1-x, x+3, 3x \rangle$$

8. (5 points) Reparametrize the following curve with respect to arc length.

$$\mathbf{r}(t) = \left(\frac{2}{t^2+1} - 1 \right) \mathbf{i} + \left(\frac{2t}{t^2+1} \right) \mathbf{j}$$

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9. (5 points) The radius of a cylindrical can with top and bottom is increasing at the rate of 4 cm/sec but its total surface area remains constant at 600π cm^2. At what rate is the height changing when the radius is 10 cm?

10. (2 points) Show that the following function is not continous at $(0, 0)$.

$$f(x, y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

11. (3 points) Show the following limit does not exist.

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz}{x^2 + y^2 + z^2}$$

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12. (6 points) Let $F(x, y, z) = xy + 2xz - y^2 + z^2$.

(a) Find the directional derivative of $F(x, y, z)$ at the point $(1, -2, 1)$ in the direction of the vector $\mathbf{v} = \langle 1, 1, 2 \rangle$.

(b) Find the maximum rate of change of $F(x, y, z)$ at the point $(1, -2, 1)$.

13. (6 points) Find and classify all critical points of the function $f(x, y) = 2x^2y - 8xy + y^2 + 5$.

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14. (12 points) Use Lagrange multipliers to find the points on the surface $x^2+xy+y^2+z^2=1$ that are closest to the origin.

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15. (12 points) Let $f(x, y) = 3 + xy - x - 2y$ and T be the closed triangular region with vertices $(1, 0)$, $(5, 0)$, and $(1, 4)$. Find the absolute maximum and absolute minimum values of f on T. Be sure to justify your answer.

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16. (5 points) Find the linearization $L(x, y)$ to $f(x, y) = 1 + x\ln(xy - 5)$ at the point $(2, 3)$ and use it to approximate $f(2.01, 2.95)$.

17. (5 points) Consider the function $f(x, y, z) = z^2$ restricted to the surface $x^2 + y^2 - z = 0$. Show the method of Lagrange multipliers only gives one candidate for an extremum. Show this candidate is where f has its minimum value on the surface and that f has no maximum on the surface.

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18. (2 points) Find and sketch the domain of the function $f(x, y) = \sqrt{1 + x - y^2}$.

19. (2 points) For $f(x, y) = \cos(x) - y$, sketch and label the level curves $z = -1$, $z = 0$, and $z = 1$.

20. (2 points) Is the following domain closed? Is it bounded?

$$\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq z \leq 4 + x + y\}$$

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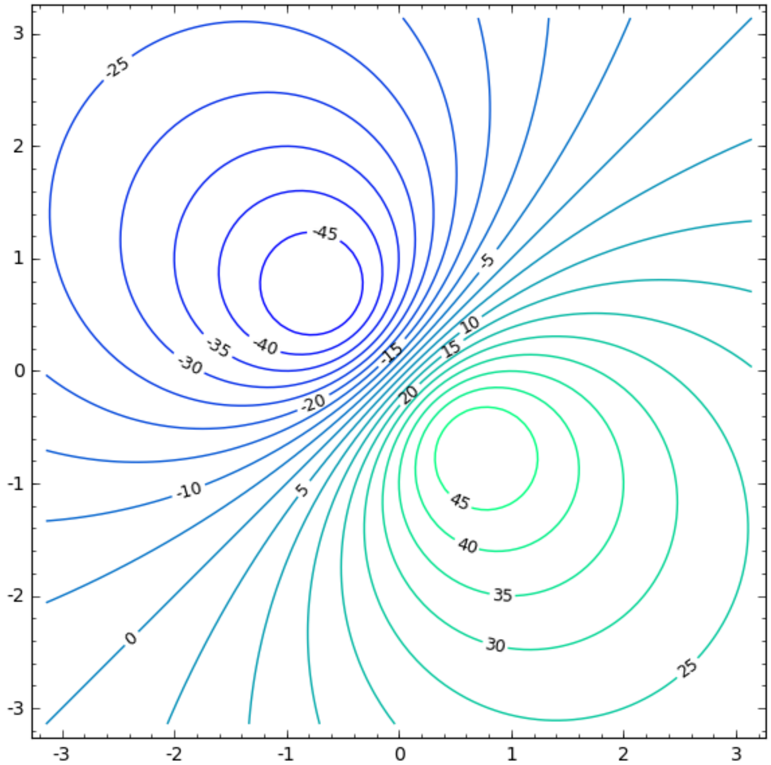
21. (10 points) Consider the contour plot for $f(x, y)$ below.



(a) If a person walked from the point $(1, -1)$ to $(1, 0)$, would they be walking uphill or downhill?

(b) If a person walked from the point $(0, 0)$ to $(1, 1)$, would they be walking uphill or downhill?

(c) Is the slope steeper at $(0, -1)$ or $(2, -2)$?

(d) Is f_y positive or negative at $(-1, 1)$?

(e) Determine the sign of each of the following derivatives.

$f_x(1, -1)$ __________ $f_y(1, -1)$ __________

$f_{xx}(1, -1)$ __________ $f_{xy}(1, -1)$ __________ $f_{yy}(1, -1)$ __________

(f) Give the components of a unit vector in the direction of ∇f at the point $(-1, 1)$. (You may estimate as necessary.)

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