Math 32A - Winter 2019
Practice Final Exam

Full Name: Solutions

UID: ______________________

Circle the name of your TA and the day of your discussion:

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Tuesday Thursday

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	10	
2	10	
3	8	
4	10	
5	12	
6	12	

Page	Points	Score
7	12	
8	10	
9	6	
10	10	
Bonus		
Total:	100	

1. (2 points) Suppose $\mathbf{u}$ is a unit vector and $\mathbf{v}$ is a vector with $||\mathbf{v}|| = 5$. If the angle θ between $\mathbf{u}$ and $\mathbf{v}$ has $\sin\theta = \frac{3}{5}$, find the length of $\mathbf{u} \times \mathbf{v}$.

$$\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\| \sin\theta$$
$$= 1 \cdot 5 \cdot \frac{3}{5}$$
$$= \boxed{3}$$

2. (3 points) Given a curve with binormal $\mathbf{B}$, show that $\dfrac{d\mathbf{B}}{ds}$ is perpendicular to $\mathbf{B}$.

$\vec{B}$ is a unit vector so $\vec{B} \cdot \vec{B} = 1$) apply $\frac{d}{ds}$

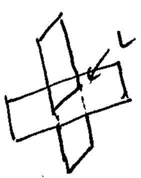
$$\frac{d\vec{B}}{ds} \cdot \vec{B} + \vec{B} \cdot \frac{d\vec{B}}{ds} = 0$$

$2 \frac{d\vec{B}}{ds} \cdot \vec{B} = 0$ so $\frac{d\vec{B}}{ds} \cdot \vec{B} = 0$ and they are $\perp$.

3. (5 points) Consider the planes $3x - 2y + z = 1$ and $2x + y - 3z = 3$, which intersect in a line L.

(a) Notice that the point $P = (1, 1, 0)$ is in the intersection of the planes and so is on L. Use P to find a vector equation for L.

$3 \cdot 1 - 2 \cdot 1 + 0 = 1$ ✓ and $2 \cdot 1 + 1 - 3 \cdot 0 = 3$ ✓ so P is on L



$\vec{n}_1 = \langle 3, -2, 1 \rangle$ $\quad \vec{n}_2 = \langle 2, 1, -3 \rangle$

If $\vec{v}$ is parallel to L then $\vec{v} \perp \vec{n}_1$ and $\vec{v} \perp \vec{n}_2$

so take $\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 1 \\ 2 & 1 & -3 \end{vmatrix} = \langle 6-1, -(-9-2), 3+4 \rangle = \langle 5, 11, 7 \rangle$

$\vec{r}(t) = \vec{r}_0 + t\vec{v} = \langle 1, 1, 0 \rangle + t \langle 5, 11, 7 \rangle$

$\boxed{\vec{r}(t) = \langle 1+5t, 1+11t, 7t \rangle}$

(b) If θ is the angle between the planes, find $\cos\theta$.

θ = angle between $\vec{n}_1$ and $\vec{n}_2$

$\vec{n}_1 \cdot \vec{n}_2 = \|\vec{n}_1\| \|\vec{n}_2\| \cos\theta$

$3 \cdot 2 - 2 \cdot 1 - 1 \cdot 3 = \sqrt{9+4+1} \sqrt{4+1+9} \cos\theta$

$1 = (9+4+1) \cos\theta = 14 \cos\theta$ so $\boxed{\cos\theta = \frac{1}{14}}$

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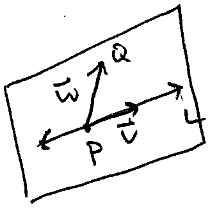
$\vec{Q}$

4. (5 points) Find the equation of the plane that passes through the point $(1,2,3)$ and contains the line given by the parametric equations $x = 3t, y = 1+t, z = 2-t$.



The line goes through point $P = (0,1,2)$ and has direction vector $\vec{v} = \langle 3,1,-1 \rangle$

To get another vector in the plane $\vec{w} = \vec{PQ} = \langle 1-0, 2-1, 3-2 \rangle$

$\vec{w} = \langle 1,1,1 \rangle$

Take $\vec{n} = \vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \langle 1+1, -(3+1), 3-1 \rangle = \langle 2,-4,2 \rangle$

so $\vec{n} = \langle 2,-4,2 \rangle$

plane: $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$ (use P or Q)

$2(x-0) - 4(y-1) + 2(z-2) = 0$

$\boxed{2x - 4y + 2z = 0}$ or $\boxed{x - 2y + z = 0}$

5. (2 points) Suppose that $w = f(x,y,z)$, $y = g(s,t)$, and $z = h(t)$. Write down the form of the chain rule you would use to compute $\partial w/\partial s$ and $\partial w/\partial t$.

$$\frac{\partial w}{\partial s} = \frac{\partial f}{\partial x}\underbrace{\cancel{\frac{\partial x}{\partial s}}}_{0} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\underbrace{\cancel{\frac{\partial z}{\partial s}}}_{0} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$$

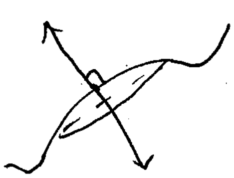
so $\boxed{\frac{\partial w}{\partial s} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}}$,

$$\frac{\partial w}{\partial t} = \frac{\partial f}{\partial x}\underbrace{\cancel{\frac{\partial x}{\partial t}}}_{0} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial t} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

really 1 variable

$\boxed{\frac{\partial w}{\partial t} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{dz}{dt}}$

6. (3 points) Find parametric equations for the line normal to the surface $\sin(xyz) = x + 2y + 3z$ at the point $(2,-1,0)$.



$\sin(xyz) = x + 2y + 3z$ $\qquad \vec{r}_0 = \langle 2,-1,0 \rangle$

$F(x,y,z) = \sin(xyz) - x - 2y - 3z = 0$ $\qquad \nabla F \perp$ surface

$\nabla F = \langle yz\cos(xyz) - 1, xz\cos(xyz) - 2, xy\cos(xyz) - 3 \rangle$

$\vec{v} = \nabla F(2,-1,0) = \langle -1, -2, -2\cos(0) - 3 \rangle = \langle -1,-2,-5 \rangle$

line: $\vec{r}(t) = \vec{r}_0 + t\vec{v} = \langle 2,-1,0 \rangle + t\langle -1,-2,-5 \rangle$

$\vec{r}(t) = \langle 2-t, -1-2t, -5t \rangle$ so $\boxed{\begin{cases} x = 2-t \\ y = -1-2t \\ z = -5t \end{cases}}$

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7. (3 points) For what values of x are the following vectors orthogonal?

$$\mathbf{v} = \langle x, x-1, x+1 \rangle \qquad \mathbf{w} = \langle 1-x, x+3, 3x \rangle$$

$$0 \stackrel{\text{set}}{=} \vec{v} \cdot \vec{w} = x(1-x) + (x-1)(x+3) + (x+1)(3x)$$

$$\cancel{x} - \cancel{x^2} + \cancel{x^2} - \cancel{x} + 3x - 3 + 3x^2 + 3x = 0$$

$$3x^2 + 6x - 3 = 0$$

$$x^2 + 2x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2(1)} = \frac{-2 \pm \overset{2\sqrt{2}}{\sqrt{8}}}{2} = -1 \pm \sqrt{2}$$

so $\boxed{x = -1 + \sqrt{2}, \; x = -1 - \sqrt{2}}$

8. (5 points) Reparametrize the following curve with respect to arc length.

$$\mathbf{r}(t) = \left(\frac{2}{t^2+1} - 1\right)\mathbf{i} + \left(\frac{2t}{t^2+1}\right)\mathbf{j} = \left\langle \frac{2}{t^2+1} - 1, \frac{2t}{t^2+1} \right\rangle$$

arc length $s(t) = \int_0^t \|\vec{r}\,'(u)\| \, du$

$$\vec{r}\,'(t) = \left\langle \frac{-4t}{(t^2+1)^2}, \frac{2(t^2+1) - (2t)(2t)}{(t^2+1)^2} \right\rangle = \left\langle \frac{-4t}{(t^2+1)^2}, \frac{2-2t^2}{(t^2+1)^2} \right\rangle$$

$$\|\vec{r}\,'(t)\| = \sqrt{\frac{16t^2}{(t^2+1)^4} + \frac{(2-2t^2)^2}{(t^2+1)^4}} = \sqrt{\frac{16t^2 + 4 - 8t^2 + 4t^4}{(t^2+1)^4}}$$

$$= \sqrt{\frac{4t^4 + 8t^2 + 4}{(t^2+1)^4}} = \sqrt{\frac{4(t^4 + 2t^2 + 1)}{(t^2+1)^4}} = 2\sqrt{\frac{(t^2+1)^2}{(t^2+1)^4}}$$

$$= 2 \cdot \frac{1}{\sqrt{(t^2+1)^2}} = \frac{2}{t^2+1}$$

so $s = \int_0^t \frac{2}{u^2+1} \, du = 2[\arctan(u)]_0^t = 2\arctan(t) - 2\overset{0}{\cancel{\arctan(0)}}$

$$s = 2\arctan(t) \longrightarrow \frac{s}{2} = \arctan(t) \longrightarrow t = \tan\left(\frac{s}{2}\right)$$

$$\boxed{\vec{q}(s) = \vec{r}(t(s)) = \left\langle \frac{2}{\tan^2(\frac{s}{2})+1} - 1, \frac{2\tan(\frac{s}{2})}{\tan^2(\frac{s}{2})+1} \right\rangle, \quad 0 \le s \le 2\pi}$$

(with some simplification this is actually $\vec{q}(s) = \langle \cos(s), \sin(s) \rangle$!)

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9. (5 points) The radius of a cylindrical can with top and bottom is increasing at the rate of 4 cm/sec but its total surface area remains constant at 600π cm^2. At what rate is the height changing when the radius is 10 cm?

$\frac{dr}{dt} = 4 \text{cm/s}$, $SA = 2\pi r^2 + 2\pi r h = 600\pi \text{ cm}^2$, $\frac{dh}{dt} = ?$ at $r = 10$ cm

$\div 2\pi$

$r^2 + rh = 300$ $\quad \frac{d}{dt}$

$2r\frac{dr}{dt} + \frac{dr}{dt}h + r\frac{dh}{dt} = 0$

$2 \cdot 10 \cdot 4 + 4h + 10\frac{dh}{dt} = 0$

$80 + 4h + 10\frac{dh}{dt} = 0$

$h = ?$ if $r = 10$

$100 + 10h = 300$

$10h = 200$

$h = 20$

so $80 + 80 + 10\frac{dh}{dt} = 0$

$\boxed{\frac{dh}{dt} = -16 \text{ cm/s}}$

10. (2 points) Show that the following function is not continous at $(0, 0)$.

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

wts $\lim_{(x,y)\to(0,0)} f(x,y) \neq f(0,0) = 0$

along $y = 0$ $\quad \lim_{x\to 0} f(x,0) = \lim_{x\to 0} \frac{x^2 - 0}{x^2 + 0} = \lim_{x \to 0} \frac{\cancel{x^2}}{\cancel{x^2}} = 1$

So even if limit exists (which it doesn't) it can't be 0,

and f is <u>not</u> continuous at $(0,0)$.

11. (3 points) Show the following limit does not exist.

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz}{x^2 + y^2 + z^2}$$

$f(x,y,z) = \frac{xy + yz}{x^2 + y^2 + z^2}$

choose paths:

along x-axis $y = 0, z = 0$

$\lim_{x\to 0} f(x,0,0) = \lim_{x\to 0} \frac{0}{x^2} = 0$

along $y = x$, $z = 0$

$\lim_{x\to 0} f(x,x,0) = \lim_{x\to 0} \frac{x^2 + 0}{x^2 + x^2} = \lim_{x\to 0} \frac{\cancel{x^2}}{2\cancel{x^2}} = \frac{1}{2}$

$0 \neq \frac{1}{2}$ so limit does not exist.

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12. (6 points) Let $F(x, y, z) = xy + 2xz - y^2 + z^2$.

(a) Find the directional derivative of $F(x, y, z)$ at the point $(1, -2, 1)$ in the direction of the vector $\mathbf{v} = \langle 1, 1, 2\rangle$.

$D_{\vec{u}}F = \nabla F \cdot \vec{u}$ $\quad \vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{1+1+4}}\langle 1,1,2\rangle = \langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\rangle$

$\nabla F = \langle y+2z, x-2y, 2x+2z\rangle$ so $\nabla F(1,-2,1) = \langle 0, 5, 4\rangle$

$D_{\vec{u}}F(1,-2,1) = \nabla F(1,-2,1)\cdot\vec{u} = \langle 0,5,4\rangle \cdot \langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\rangle = \frac{5}{\sqrt{6}} + \frac{8}{\sqrt{6}} = \boxed{\frac{13}{\sqrt{6}}}$

(b) Find the maximum rate of change of $F(x, y, z)$ at the point $(1, -2, 1)$.

max rate of change is $\|\nabla F\|$

so $\|\langle 0,5,4\rangle\| = \sqrt{25+16} = \boxed{\sqrt{41}}$

13. (6 points) Find and classify all critical points of the function $f(x, y) = 2x^2y - 8xy + y^2 + 5$.

1. critical pts: $\left.\begin{array}{l} f_x = 4xy - 8y \overset{\text{set}}{=} 0 \\ f_y = 2x^2 - 8x + 2y \overset{\text{set}}{=} 0 \end{array}\right\}$ $\quad 4y(x-2) = 0$ so $y = 0$ or $x = 2$

if $y = 0$: $2x^2 - 8x = 0$, $2x(x-4) = 0$, $x = 0, 4$ so critical pts $(0,0), (4,0)$

if $x = 2$: $8 - 16 + 2y = 0$, $2y = 8$, $y = 4$ so critical pt $(2,4)$

2. 2nd partials test $D = f_{xx}f_{yy} - f_{xy}^2$

$f_{xx} = 4y$

$f_{yy} = 2$

$f_{xy} = 4x - 8 = f_{yx}$

$D = (4y)(2) - (4x-8)^2 = 8y - (4x-8)^2$

At $(0,0)$: $D(0,0) = -64 < 0$ so saddle pt

At $(4,0)$: $D(4,0) = -64 < 0$ so saddle pt

At $(2,4)$: $D(2,4) = 32 > 0$, $f_{xx}(2,4) > 0$ so local min

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14. (12 points) Use Lagrange multipliers to find the points on the surface $x^2+xy+y^2+z^2=1$ that are closest to the origin.

↑ min. distance

↑ ellipsoid
closed and bounded
So EVT holds ✓

Instead minimize $d^2 = f(x,y,z) = x^2+y^2+z^2$

subject to constraint $g(x,y,z) = x^2+xy+y^2+z^2 = 1$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = k \end{cases} \qquad \begin{cases} 2x = \lambda(2x+y) \\ 2y = \lambda(x+2y) \\ 2z = \lambda(2z) \\ x^2+xy+y^2+z^2 = 1 \end{cases}$$

$2z = \lambda(2z) \longrightarrow 2z - \lambda(2z) = 0$

$2z(1-\lambda) = 0$

So $z=0$ or $\lambda = 1$

if $\lambda = 1$:
$$\begin{cases} 2x = 2x+y \longrightarrow y=0 \\ 2y = x+2y \longrightarrow x=0 \\ 2z = 2z \\ x^2+xy+y^2+z^2=1 \leadsto z^2=1, \text{ so } z=\pm 1 \end{cases}$$

consider $(0,0,1)$ and $(0,0,-1)$

if $\lambda \neq 1$ then $z=0$:
$$\begin{cases} 2x = \lambda(2x+y) & \cdot y \\ 2y = \lambda(x+2y) & \cdot x \\ 0 = 0 \\ x^2+xy+y^2 = 1 \end{cases} \Rightarrow \begin{cases} 2xy = \lambda 2xy + \lambda y^2 \\ 2xy = \lambda x^2 + \lambda 2xy \end{cases}$$

So $\cancel{\lambda 2xy} + \lambda y^2 = \lambda x^2 + \cancel{\lambda 2xy}$

$\lambda y^2 - \lambda x^2 = 0$

$\lambda(y^2-x^2) = 0 \longrightarrow \lambda = 0$ or $y^2 = x^2$

↳ if $\lambda = 0$: $x=0$, $y=0$, $z=0$
but this doesn't satisfy constraint X

↳ if $\lambda \neq 0$ then $y^2 = x^2$:

If $y = x$: $3x^2 = 1$ So $x = \pm\frac{1}{\sqrt{3}}$, $y=x$

If $y=-x$: $x^2 = 1$ So $x = \pm 1$, $y=-x$

$f(0,0,1) = 1$

$f(0,0,-1) = 1$

$f(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, 0) = \frac{2}{3}$

$f(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, 0) = \frac{2}{3}$ > min

$f(1,-1,0) = 2$

$f(-1,1,0) = 2$

↳ closest points are $\boxed{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, 0\right) \text{ and } \left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, 0\right)}$

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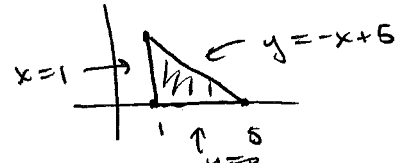

15. (12 points) Let $f(x,y) = 3 + xy - x - 2y$ and T be the closed triangular region with vertices $(1,0)$, $(5,0)$, and $(1,4)$. Find the absolute maximum and absolute minimum values of f on T. Be sure to justify your answer.

(Guaranteed global extrema by Extreme Value Thm b/c region is closed and bounded)

1. critical pts: $f_x = y - 1 \overset{\text{set}}{=} 0$, $f_y = x - 2 \overset{\text{set}}{=} 0$ $\Big\}$ $y = 1$, $x = 2$ $\;$ $(2,1)$ critical pt inside T ✓

2. boundary pts:

① $y = 0$, $1 \le x \le 5$

$g(x) = f(x,0) = 3 - x$, $1 \le x \le 5$

$g'(x) = -1 \ne 0$ so no critical pts

end pts: $g(1) = f(1,0)$

$g(5) = f(5,0)$

② $x = 1$, $0 \le y \le 4$

$h(y) = f(1,y) = 3 + y - 1 - 2y = 2 - y$, $0 \le y \le 4$

$h'(y) = -1 \ne 0$ so no critical pts

end pts: $h(0) = f(1,0)$ ✓

$h(4) = f(1,4)$

③ $y = -x + 5$, $1 \le x \le 5$

$k(x) = f(x, -x+5) = 3 + x(-x+5) - x - 2(-x+5) = 3 - x^2 + 5x - x + 2x - 10$

$k(x) = -x^2 + 6x - 7$, $1 \le x \le 5$

$k'(x) = -2x + 6 \overset{\text{set}}{=} 0$ $\quad x = 3$ so $k(3) = f(3,2)$

end pts: $k(1) = f(1,4)$ ✓

$k(5) = f(5,0)$ ✓

compare: $f(2,1) = 3 + \cancel{2} - \cancel{2} - 2 = 1$

$f(1,0) = 3 - 1 = 2$

$f(5,0) = 3 - 5 = -2$

$f(1,4) = 3 + 4 - 1 - 8 = -2$

$f(3,2) = 3 + 6 - 3 - 4 = 2$

So $\boxed{\begin{array}{l} \text{abs. max is } 2 \ (\text{at } (1,0) \text{ and } (3,2)) \\ \text{abs. min is } -2 \ (\text{at } (5,0) \text{ and } (1,4)) \end{array}}$

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16. (5 points) Find the linearization $L(x, y)$ to $f(x, y) = 1 + x\ln(xy - 5)$ at the point $(2, 3)$ and use it to approximate $f(2.01, 2.95)$.

near (a,b) $\quad f(x,y) \approx L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$f(2,3) = 1 + 2\ln(6-5) = 1 + 2\underbrace{\ln(1)}_{0} = 1$

$f_x = \ln(xy-5) + \dfrac{xy}{xy-5} \qquad f_x(2,3) = \underbrace{\ln(1)}_{0} + \frac{6}{1} = 6$

$f_y = \dfrac{x^2}{xy-5} \qquad f_y(2,3) = \frac{4}{1} = 4$

so near $(2,3)$ $\quad f(x,y) \approx L(x,y) = f(2,3) + f_x(2,3)(x-2) + f_y(2,3)(y-3)$

$L(x,y) = 1 + 6(x-2) + 4(y-3)$

$f(2.01, 2.95) \approx L(2.01, 2.95) = 1 + 6(0.01) + 4(-0.05) = 0.86$

So $\boxed{f(2.01, 2.95) \approx 0.86}$

17. (5 points) Consider the function $f(x, y, z) = z^2$ restricted to the surface $x^2 + y^2 - z = 0$. Show the method of Lagrange multipliers only gives one candidate for an extremum. Show this candidate is where f has its minimum value on the surface and that f has no maximum on the surface.

$f(x,y,z) = z^2$ subject to constraint $g(x,y,z) = x^2+y^2-z = 0$

$z = x^2+y^2$ circular paraboloid
not bounded.
EVT doesn't apply!

$\begin{cases} \nabla f = \lambda \nabla g \\ g = k \end{cases}$

$\begin{cases} 0 = \lambda(2x) \to x = 0 \text{ or } \lambda = 0 \\ 0 = \lambda(2y) \to y = 0 \text{ or } \lambda = 0 \\ 2z = \lambda(-1) \\ x^2+y^2-z = 0 \end{cases}$

if $\lambda = 0$: then $z = 0$ so $x^2+y^2 = 0 \Rightarrow x=0, y=0$
so $(0,0,0)$ is only candidate.

if $\lambda \neq 0$ then $x=0$ and $y=0$: $-z = 0$ so
again $(0,0,0)$ is only candidate.

But $f(x,y,z) = z^2 \geq 0$ and $f(0,0,0) = 0$ so this the minimum

Set $y=0 \to z = x^2$ so $f(x,0,x^2) = x^4$ and $\lim_{x\to\infty} x^4 = \infty$ so no max!

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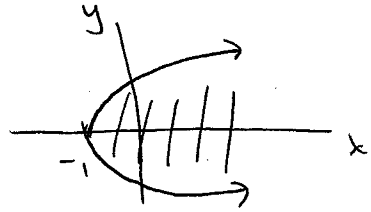
18. (2 points) Find and sketch the domain of the function $f(x,y) = \sqrt{1+x-y^2}$.

$1 + x - y^2 \geq 0$

$1 + x \geq y^2$

$y^2 = 1 + x$ parabola

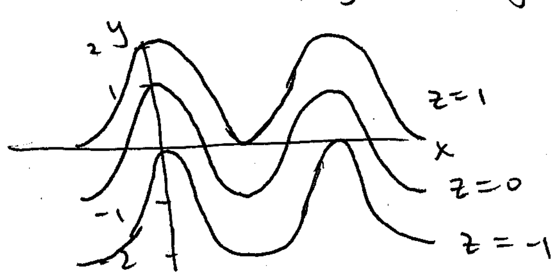


19. (2 points) For $f(x,y) = \cos(x) - y$, sketch and label the level curves $z=-1$, $z=0$, and $z=1$.

$z = -1$: $-1 = \cos(x) - y$ so $y = \cos(x) + 1$

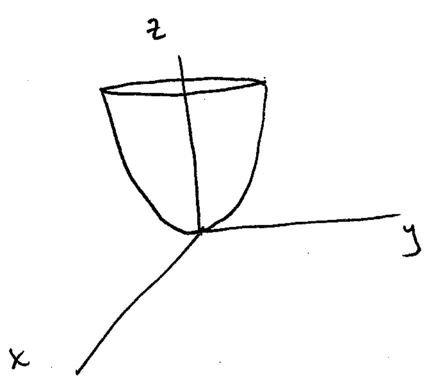
$z = 0$: $0 = \cos(x) - y$ so $y = \cos(x)$

$z = 1$: $1 = \cos(x) - y$ so $y = \cos(x) - 1$



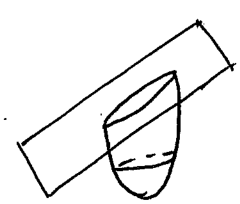
20. (2 points) Is the following domain closed? Is it bounded?

$$\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 \leq z \leq 4 + x + y\}$$



$z = x^2 + y^2$ circular paraboloid

$z = 4 + x + y$ plane



yes closed and bounded

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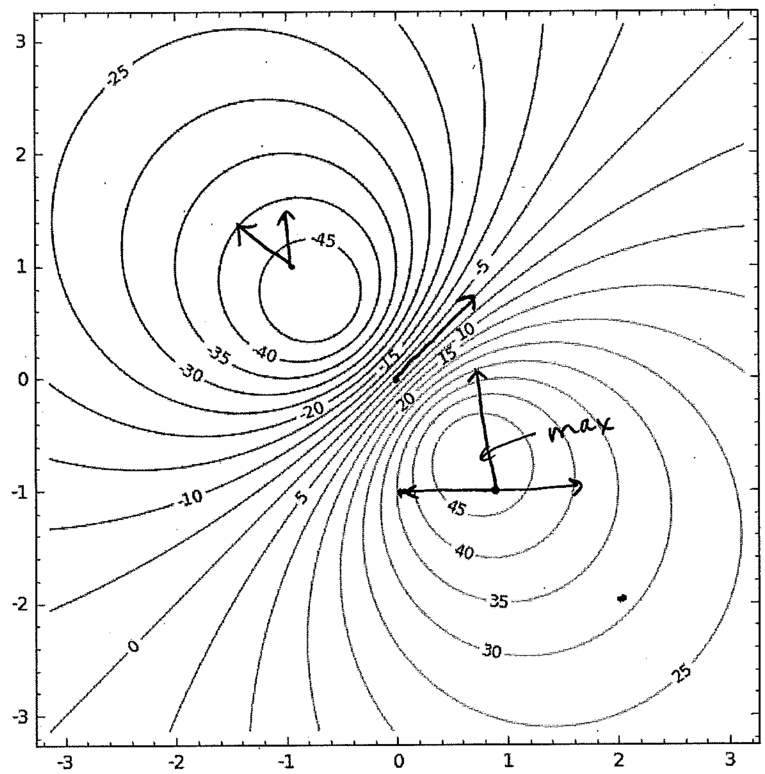
21. (10 points) Consider the contour plot for $f(x, y)$ below.



(a) If a person walked from the point $(1, -1)$ to $(1, 0)$, would they be walking uphill or downhill?

downhill

(b) If a person walked from the point $(0, 0)$ to $(1, 1)$, would they be walking uphill or downhill?

neither - walking along a level curve

(c) Is the slope steeper at $(0, -1)$ or $(2, -2)$?

At $(0,-1)$ - level curves are closer together

(d) Is f_y positive or negative at $(-1, 1)$?

positive

(e) Determine the sign of each of the following derivatives.

$f_x(1, -1)$ $\underline{\quad - \quad}$ $f_y(1, -1)$ $\underline{\quad + \quad}$

$f_{xx}(1, -1)$ $\underline{\quad - \quad}$ $f_{xy}(1, -1)$ $\underline{\quad - \quad}$ $f_{yy}(1, -1)$ $\underline{\quad - \quad}$

(f) Give the components of a unit vector in the direction of ∇f at the point $(-1, 1)$. (You may estimate as necessary.)

direction of $\langle -1, 1 \rangle$ so $\boxed{\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle}$

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