1. Find and classify all critical points of the function

$$f(x, y) = 3 - 2x^4 + 5x^2 - 2xy + y^2.$$

2. Find the global extreme values of the function

$$f(x, y) = x^2 + 4y - 4xy + 2$$

over the region bounded by $y = x$, $y = 0$, and $x = 2$.

3. If a function of one variable is continuous on an interval and has only one critical point, then a local maximum has to be a global maximum. This is not true for functions of two variables. Show that the function

$$f(x, y) = 3xe^y - x^3 - e^{3y}$$

has exactly one critical point. Show that f has a local maximum at this critical point but that f has no global maximum. Use a computer to graph the function to see how this is possible.

4. Consider the function $f(x, y) = x^3 - 3xy^2$.

   (a) Show that f has only one critical point and that at this point the second partials test is inconclusive.

   (b) Classify the critical point another way. (*Hint:* Consider restricting the function to the x-axis.)

   (c) Use a computer to graph the function and check your answer to part (b). The graph is called a *monkey saddle*.