

1. Assume that $\|\mathbf{v}\| = 2$ and $\|\mathbf{w}\| = 3$, and that the angle between \mathbf{v} and \mathbf{w} is $\frac{2\pi}{3}$.

(a) Find $\mathbf{v} \cdot \mathbf{w}$.

(b) Find $\|\mathbf{v} + 2\mathbf{w}\|$.

(c) Find $\|\mathbf{v} - 2\mathbf{w}\|$.

2. Show that if $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are orthogonal, then the vectors \mathbf{u} and \mathbf{v} must have the same magnitude.

3. Consider the vectors $\mathbf{u} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$, and $\mathbf{w} = 5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$.

(a) Use the equation of a plane to show the three vectors are coplanar.

(b) Use the scalar triple product to show the three vectors are coplanar.

4. Consider the sphere with radius 4 and center $(7, -2, -1)$. Find the point on the sphere that is closest to the plane $2x - 3y - z = -7$.

5. Find the equation of the plane that contains the line $x = 3 + 2t$, $y = t$, $z = 8 - t$ and is parallel to the plane $2x + 4y + 8z = 17$.