

1. Consider the curve $\mathbf{r}(t) = \langle \cos t, \sqrt{2} \sin t, \cos t \rangle$.

(a) Find the length of the curve $\mathbf{r}(t)$ for $0 \leq t \leq 2\pi$.

(b) Find an arc length parametrization of the space curve parametrized by $\mathbf{r}(t)$.

(c) Find the unit tangent $\mathbf{T}(t)$.

(d) Find $\mathbf{T}'(t)$ and $\|\mathbf{T}'(t)\|$.

2. Consider the curve $\mathbf{r}(t) = \langle 2t^3 + 4, 2t^3, 3t^2 - 7 \rangle$.
- (a) Find the length of the curve $\mathbf{r}(t)$ for $0 \leq t \leq 3$.

(b) Find the unit tangent $\mathbf{T}(t)$.

(c) Find $\mathbf{T}'(t)$ and $\|\mathbf{T}'(t)\|$.

3. Find a parametrization of a path that traces the circle in the plane $y = 10$ with radius 3 and center $(1, 10, -3)$ with constant speed 6.

4. In this problem you will prove that if $\|\mathbf{r}(t)\| = c$ is a constant then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$ for all t .

(a) Write an equation that gives $\mathbf{r}(t) \cdot \mathbf{r}(t)$ in terms of c .

(b) Apply $\frac{d}{dt}$ to both sides of your equation from part (a).

(c) Simplify the equation from part (b) as much as possible to conclude that for all t , $\mathbf{r}'(t) \cdot \mathbf{r}(t) = 0$. This completes the proof.

(d) As a consequence, show the work above implies $\mathbf{T}(t)$ is orthogonal to $\mathbf{T}'(t)$.