Math 32A, Winter 2019 **WS - Week 8** **Full Name:**

---

1. Let $f(x) = 5x$. In this problem you will show that $\lim_{x \to 1} f(x) = 5$.

   (a) Write the definition of the limit in this context using ε and δ.

   (b) Let $\varepsilon = 1$. How small does δ have to be so that if $|x - 1| < \delta$ then $|5x - 5| < 1$? (*Hint:* Factor out a 5.)

   (c) Let $\varepsilon = \frac{1}{2}$. How small does δ have to be so that if $|x - 1| < \delta$ then $|5x - 5| < \frac{1}{2}$?

   (d) Now let $\varepsilon > 0$ be arbitrary. Then choose an appropriate δ (which will be an expression involving ε). Show that if $0 < |x - 1| < \delta$ for your choice of δ, then $|5x - 5| < \varepsilon$.

2. Either give an example of a function $f(x, y)$ with partial derivatives $f_x(x, y) = 2x + y^2 \cos x$ and $f_y(x, y) = x^2 + y^2 \sin x$ or show that no such function f can exist.

3. Use the chain rule to calculate $\frac{d}{dt} f(\mathbf{r}(t))$ if $f(x, y) = 3 \ln(x) + \ln(y)$ and $\mathbf{r}(t) = \langle \cos t, t^2 \rangle$ at $t = \frac{\pi}{4}$.

4. Find the directional derivative of $f(x, y, z) = xyz + z^3$ at the point $P = (-3, 2, -1)$ in the direction pointing to the origin.

5. Consider the function $f(x, y) = e^{xy-y^2}$.

(a) Use a linear approximation to $f(x, y)$ at the point $(1, 1)$ to estimate the value of $f(1.02, 1.01)$.

(b) Find the directional derivative of f at the point $(1, 1)$ in the direction of $\langle 3, 4 \rangle$.

(c) Find the maximum rate of change of f at the point $(1, 1)$.