**Math 32A, Winter 2019** **WS - Week 9** **Full Name:**

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1. Let $f(x,y) = \dfrac{2xy^2}{x^2+y^2}$. In this problem you will show that $\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

   (a) Write the definition of the limit in this context using $\varepsilon$ and $\delta$.

   (b) For the scratch work, we will start with the expression $\left|\dfrac{2xy^2}{x^2+y^2} - 0\right|$ and try to relate it to $\sqrt{x^2+y^2}$. In the first expression, replace $x$ in the numerator with $\sqrt{x^2}$. Why is this valid?

   (c) Now replace the term $\sqrt{x^2}$ with $\sqrt{x^2+y^2}$. How is this related to the previous expression? (*Hint:* Write an inequality.)

   (d) Next replace $y^2$ in the numerator (the term not under the square root) with $x^2+y^2$. How is this related to the previous expression?

   (e) Simplify the expression above. You should end up with a number multiplied by $\sqrt{x^2+y^2}$. The coefficient should tell you how to choose $\delta$ from $\varepsilon$.

   (f) Now let $\varepsilon > 0$ be arbitrary. Then choose an appropriate $\delta$ (which will be an expression involving $\varepsilon$). Show that if $0 < \sqrt{x^2+y^2} < \delta$ for your choice of $\delta$, then indeed $\left|\dfrac{2xy^2}{x^2+y^2} - 0\right| < \varepsilon$.

2. Let $f(x, y, z) = xyz - z^2$, where $x = r\cos\theta$, $y = r\sin\theta$, and $z = r$. Use the chain rule to calculate the partial derivatives $\dfrac{\partial f}{\partial \theta}$ and $\dfrac{\partial f}{\partial r}$.

3. Let $x = 4s + t$ and $y = 4s - t$. Show that for any differentiable function $f(x, y)$,

$$\left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2 = \frac{1}{4}\,\frac{\partial f}{\partial s}\,\frac{\partial f}{\partial t}.$$

4. Consider the surface defined by $\sin(xyz) = x + 2y + 3z$.

(a) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ by first applying the differential operators $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$.

(b) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ by first writing $F(x, y, z) = 0$.

(c) Check that your answers in parts (a) and (b) agree.