Math 32B - Fall 2019
Practice Exam 2

Full Name: ______________________________

UID: ______________________________

Circle the name of your TA and the day of your discussion:

Steven Gagniere   Jason Snyder   Ryan Wilkinson

Tuesday   Thursday

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. (20 points) Let $\mathbf{F}(x, y, z) = \langle e^y, xe^y, (z+1)e^z \rangle$ and let $\mathcal{C}$ be the curve parameterized by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ for $0 \leq t \leq 1$.

(a) Show that the vector field $\mathbf{F}$ is conservative.

(b) Find a potential function for $\mathbf{F}$.

(c) Use parts (a) and (b) to evaluate $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

(d) Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that $\operatorname{curl} \mathbf{G} = \mathbf{F}$?

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2. (10 points) Evaluate the line integral $\int_{\mathcal{C}} (x^2+y^2+z^2)\, ds$ where $\mathcal{C}$ is the helix parameterized by $x = t,\ y = \cos 2t,\ z = \sin 2t$ for $0 \le t \le 2\pi$.

3. (10 points) Evaluate the line integral $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = \langle 2y + z, x - 3z, x + y \rangle$ and $\mathcal{C}$ is the line segment from $(1, 0, 2)$ to $(2, 3, -1)$.

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4. (20 points) The velocity field of a fluid is given by $\mathbf{F}(x, y, z) = \langle x, y, z^4 \rangle$. Find the flux of the fluid across the closed surface given by $z^2 = x^2 + y^2$ for $0 \leq z \leq 1$ and $x^2 + y^2 \leq 1$ at $z = 1$ with positive orientation.

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5. (20 points) Let S be a portion of the helicoid parameterized by

$$\mathbf{r}(u,v) = \langle u\cos v, u\sin v, v\rangle \qquad \text{for} \quad 0 \le u \le 1, \quad 0 \le v \le \pi.$$

(a) Compute $\displaystyle\iint_S 2y\,dS$.

(b) Let $\mathbf{F}(x,y,z) = \langle x,y,z\rangle$ and compute $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S}$.

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6. (10 points) Let $\mathbf{F}(x, y) = \langle y^2 \cos x, x^2 + 2y \sin x \rangle$ and let $\mathcal{C}$ be the path along the triangle from $(0, 0)$ to $(2, 6)$ to $(2, 0)$ and back to $(0, 0)$. Use Green's Theorem to evaluate $\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

7. (10 points) Use Green's Theorem to find the area of the annulus $\mathcal{R}$ bounded by two circles centered at the origin, one with radius 3 and the other with radius 5. (You should be able to check your answer easily by computing the area another way).

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