Math 32B - Fall 2019
Practice Final Exam

Full Name: ______________________________

UID: ______________________________

Circle the name of your TA and the day of your discussion:

Steven Gagniere   Jason Snyder   Ryan Wilkinson

Tuesday   Thursday

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	10	
2	10	
3	10	
4	10	
5	15	

Page	Points	Score
6	15	
7	15	
8	15	
Bonus		
Total:	100	

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1. (5 points) Let $\mathcal{R}$ be the trapezoid with vertices $(1,0)$, $(2,0)$, $(0,2)$, and $(0,1)$. Evaluate the iterated integral $\displaystyle\iint_{\mathcal{R}} \cos\left(\frac{y-x}{y+x}\right)\, dA$.

2. (5 points) Find the volume of the solid that lies above the xy-plane, within the sphere $x^2+y^2+z^2=4$ and below the cone $z=2\sqrt{x^2+y^2}$.

/10

3. (10 points) Find the surface area of the portion $\mathcal{S}$ of $z = \sqrt{x^2 + y^2}$ contained within the cylinder $x^2 + z^2 \leq 1$.

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4. (5 points) Evaluate the line integral $\int_{\mathcal{C}} y \sin z \, ds$ where $\mathcal{C}$ is parameterized by $x = \cos t$, $y = \sin t$, $z = t$ for $0 \leq t \leq 2\pi$.

5. (5 points) Let $\mathbf{F}(x, y, z) = \langle 2xy^2 \cos z, 2x^2 y \cos z + 2y, -x^2 y^2 \sin z + 1 \rangle$. Find the work done by the vector field $\mathbf{F}$ in moving a particle along the curve $\mathcal{C}$ parameterized by $\mathbf{r}(t) = \langle t, \sin t, t^2 + 1 \rangle$ for $0 \leq t \leq \pi$

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6. (5 points) Find the average value of $f(x, y, z) = xy^2z^3$ on the box $[0, 1] \times [0, 2] \times [0, 3]$.

7. (5 points) The Laplace operator Δ of a function $f(x, y, z)$ is defined by

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}.$$

A function f satisfying $\Delta f = 0$ is called harmonic.

(a) Show that $\mathbf{F}(x, y, z) = \left\langle xz, -yz, \frac{1}{2}(x^2 - y^2) \right\rangle$ is the gradient of a harmonic function.

(b) Find the flux of $\mathbf{F}$ through the surface $\mathcal{S}$ given by $x^2 + y^2 + z^2 = 1$ with outward normal.

/10

8. (15 points) Let $\mathbf{F}(x, y, z) = \langle xy^2 + e^{y+\cos y}, x^2 y + \sin z, z^2 + \cos x\rangle$ and let E be the solid cone consisting of the points above $z = \sqrt{x^2 + y^2}$ and below $z = 4$. Find $\displaystyle\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ where S is the surface of E with positive orientation.

/15

9. (15 points) Let $\mathbf{F}(x, y, z) = \langle x, x+y^3, x^2+y^2-z \rangle$ and let S be the surface $z = x^2 - y^2$ for $x^2 + y^2 \leq 1$ with upward orientation and positively oriented boundary C. Find $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$.

/15

10. (15 points) Find $\int_0^2 \int_{x-x^2}^{2-x^2} 6x \cos\left((x^2+y)^3\right) \, dy \, dx$ using the substitutions $u = x^2 + y$ and $v = x$.

/15

11. (15 points) Let $\mathbf{F}(x, y) = \langle xy + \cos(x^2), x - \arctan(y^2)\rangle$ and let D be the region of the plane above the x-axis inside the circle centered at the origin with radius 2 and outside the circle centered at $(1, 0)$ with radius 1. Let C be the boundary of D with positive orientation. Evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$.

/15