1. Let $\mathcal{W}$ be the solid region bounded by the xy-plane and $z = 4 - x^2 - y^2$. Let $\mathcal{S}$ be the boundary surface of $\mathcal{W}$ with positive orientation and let

$$\mathbf{F}(x, y, z) = \left\langle xz\sin(yz) + x^3, \cos(yz), 3y^2 z - e^{x^2+y^2} \right\rangle.$$

Find $\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$.

2. Consider the surface $\mathcal{S}_1$ given by $z = 4 - x^2 - y^2$ for $z \geq 0$ and let

$$\mathbf{F}(x, y, z) = \left\langle xz\sin(yz) + x^3, \cos(yz), 3y^2 z - e^{x^2+y^2} \right\rangle.$$

Compute $\iint_{\mathcal{S}_1} \mathbf{F} \cdot d\mathbf{S}$. (*Hint:* Use your result from the previous problem.)

3. Let $\mathcal{W}$ be a simple solid with piecewise smooth boundary $\mathcal{S}$.

(a) What geometric quantity is computed by

$$\frac{1}{3}\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$?

(b) Are there other vector fields $\mathbf{F}(x, y, z)$ that compute the same quantity as described in part (a)? If not, explain why not. If so, give some examples.

(c) Let $\mathcal{S}$ be the sphere centered at the origin with radius R and outward pointing normal and let

$$\mathbf{F}(x, y, z) = \left\langle x - 3y^2 z, e^{xz} - y, 2z - \cos(xy) \right\rangle.$$

Use the Divergence Theorem to find the flux of $\mathbf{F}$ across $\mathcal{S}$.

4. Let $\mathcal{W}$ be part of the cone $x^2 + y^2 = (2 - z)^2$ for $0 \leq z \leq 1$. Use the Divergence Theorem to find the volume of $\mathcal{W}$. (*Hint:* You have a choice of $\mathbf{F}$. Since the boundary of $\mathcal{W}$ would normally have three pieces, make a choice of $\mathbf{F}$ so that $\mathbf{F} \cdot \mathbf{n} = 0$ on two of those pieces.)