1. Match each double integral in polar coordinates with the graph of the region of integration.

   (a) $\displaystyle\int_3^4 \int_{3\pi/4}^{7\pi/4} f(r,\theta)\, r\, d\theta\, dr$ ____________

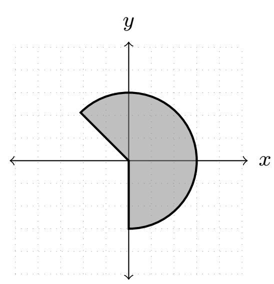
   (b) $\displaystyle\int_{3\pi/2}^{2\pi} \int_0^4 f(r,\theta)\, r\, dr\, d\theta$ ____________

   (c) $\displaystyle\int_0^3 \int_{-\pi/2}^{3\pi/4} f(r,\theta)\, r\, d\theta\, dr$ ____________

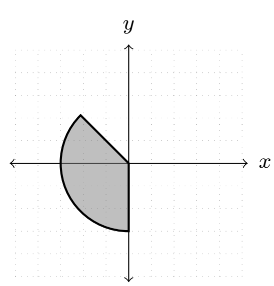
   (d) $\displaystyle\int_{3\pi/4}^{3\pi/2} \int_0^3 f(r,\theta)\, r\, dr\, d\theta$ ____________

   (e) $\displaystyle\int_0^{2\pi} \int_3^4 f(r,\theta)\, r\, dr\, d\theta$ ____________

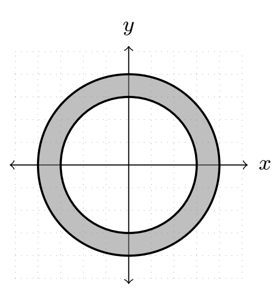
   (f) $\displaystyle\int_{-\pi/4}^{3\pi/4} \int_3^4 f(r,\theta)\, r\, dr\, d\theta$ ____________



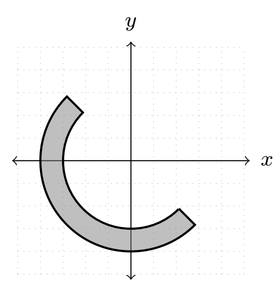
(A)



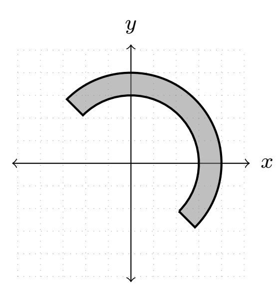
(B)



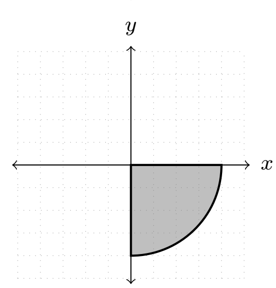
(C)



(D)



(E)



(F)

2. Find the area of the region which lies inside both the circle $r = 8\cos(\theta)$ and the circle $r = 8\sin(\theta)$.

3. Use a double integral to find the area of one loop of the rose $r = 2\cos(3\theta)$.

4. We can define an improper integral over the entire plane $\mathbb{R}^2$ in several equivalent ways. If D_a is the disk of radius a centered at the origin and S_a is the square with vertices $(\pm a, \pm a)$ then

$$\iint_{\mathbb{R}^2} f(x,y)\,dA = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx = \lim_{a\to\infty}\iint_{D_a} f(x,y)\,dA = \lim_{a\to\infty}\iint_{S_a} f(x,y)\,dA.$$

We will use this to compute

$$\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi},$$

an important integral for probability and statistics.

(a) Consider the solid under the graph of $z = e^{-x^2-y^2}$ above the disk D_a. Set up a double integral to find the volume of the solid.

(b) Evaluate the integral above and find the volume. Your answer will be in terms of a.

(c) What does the volume approach as $a \to \infty$?

(d) Now use the volume in part (c) and the interpretation of the improper integral involving S_a to find

$$\left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)^2$$

and then take the square root.

(e) Finally, making the change of variable $t = \sqrt{2}\,x$, show that

$$\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}.$$