Math 32B, Fall 2019 **WS - Week 5** **Full Name:**

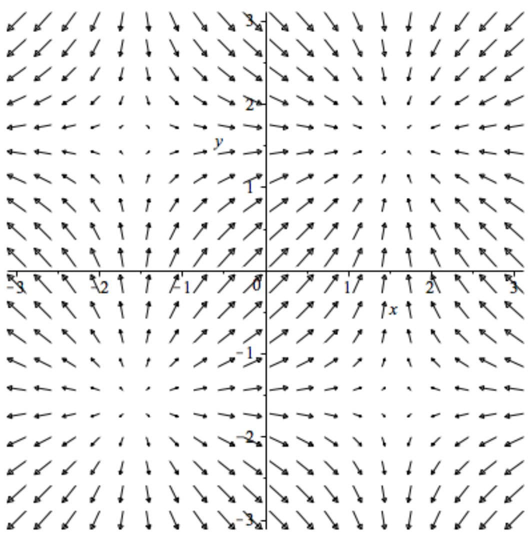
1. Match each vector field $\mathbf{F}$ with its plot below.

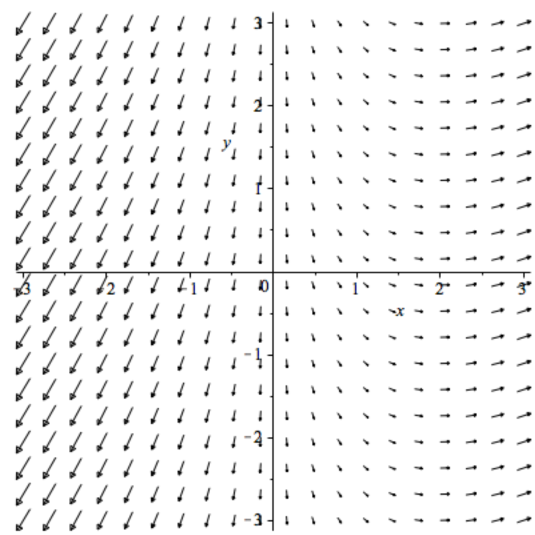
   (a) $\mathbf{F}(x, y) = \langle x, -y \rangle$

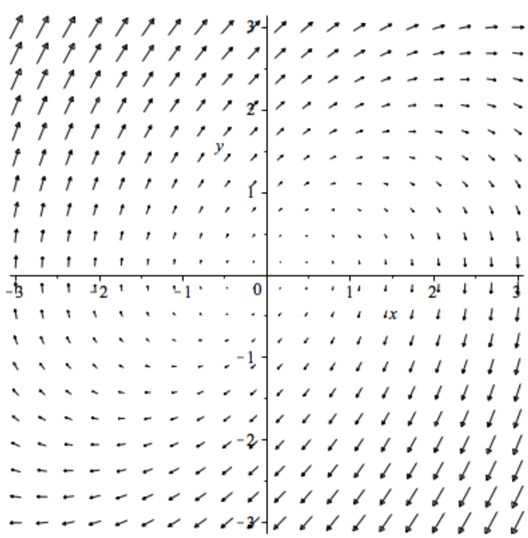
   (b) $\mathbf{F}(x, y) = \langle y, y - x \rangle$

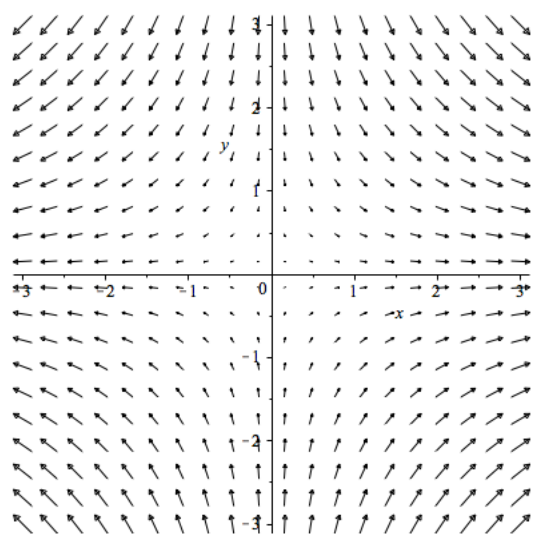
   (c) $\mathbf{F}(x, y) = \langle \cos x, \cos y \rangle$

   (d) $\mathbf{F}(x, y) = \langle x, x - 2 \rangle$

(A) 

(B) 

(C) 

(D) 

2. Which of the above vector fields are conservative? Find a potential function for each conservative vector field.

A **flow line** for a vector field $\mathbf{F}$ is a parameterized path $\mathbf{r}(t)$ such that for every t, $\mathbf{r}'(t) = \mathbf{F}(\mathbf{r}(t))$. That is, the velocity vectors for a particle traveling along the path coincide with the of the vector field at all points along the path.

3. Consider the vector fields

$$\mathbf{F}(x,y) = \frac{x}{\sqrt{x^2+y^2}}\mathbf{i} + \frac{y}{\sqrt{x^2+y^2}}\mathbf{j} \text{ and } \mathbf{G}(x,y) = \frac{-y}{\sqrt{x^2+y^2}}\mathbf{i} + \frac{x}{\sqrt{x^2+y^2}}\mathbf{j}$$

(a) Show that $\mathbf{F}(x,y)$ and $\mathbf{G}(x,y)$ have unit length for all (x,y) where they are defined.

(b) Show that $\mathbf{F}(x,y)$ is perpendicular to $\mathbf{G}(x,y)$ for all (x,y) where they are defined.

(c) Sketch the vector fields $\mathbf{F}(x,y)$ and $\mathbf{G}(x,y)$.

(d) Determine whether $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$ is a flow line for $\mathbf{F}$.

(e) Determine whether $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$ is a flow line for $\mathbf{G}$.

(f) Determine whether $\mathbf{r}(t) = \langle 2\cos t, 2\sin t \rangle$ is a flow line for $\mathbf{G}$.

4. Let $\mathbf{F}(x, y, z)$ be a conservative vector field so that $\mathbf{F} = \nabla f$ for some $f(x, y, z)$. Assume that all partial derivatives of $f(x, y, z)$ exist and are continuous. Show that
$$\operatorname{curl}(\mathbf{F}) = \operatorname{curl}(\nabla f) = \mathbf{0}.$$

5. Let $\mathbf{F}(x, y, z)$ be a vector field. Assume that all partial derivatives of $\mathbf{F}$ exist and are continuous. Show that $\operatorname{div}(\operatorname{curl}\mathbf{F}) = 0$.

6. Determine whether or not the vector field $\mathbf{F}(x, y, z) = \langle y^2, 2xy + e^{3z}, 3ye^{3z} \rangle$ is conservative. If not, explain why not. If $\mathbf{F}$ is conservative, find a potential function for $\mathbf{F}$.