1. Let $\mathbf{F}(x, y) = (y^2 + 1)\mathbf{i} + (2xy - 2)\mathbf{j}$. Compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ explicitly by parametrizing $\mathcal{C}$ where

   (a) $\mathcal{C}$ is the line segment from $(0, 0)$ to $(1, 1)$.

   (b) $\mathcal{C}$ is the path from $(0, 0)$ to $(1, 1)$ that first moves along a straight line in the positive y-direction and then along a straight line in the positive x-direction.

   (c) $\mathcal{C}$ is the path from $(0, 0)$ to $(1, 1)$ along the parabola $y = x^2$.

   (d) $\mathcal{C}$ is the arc of the circle centered at $(1, 0)$ with radius 1 from $(0, 0)$ to $(1, 1)$.

   (e) Do your answers above agree with the fundamental theorem of line integrals? Why or why not?

2. Let $f(x, y) = \sin x + x^2 y$ and $\mathbf{F} = \nabla f$. Let $\mathcal{C}$ be the curve in $\mathbb{R}^2$ parameterized by $\mathbf{r}(t) = \langle t, t^2 \rangle$ for $0 \leq t \leq \pi$.

(a) Compute the line integral $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ explicitly using the parametrization for $\mathcal{C}$.

(b) Use the fundamental theorem for line integrals to compute $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

(c) Do your answers to parts (a) and (b) agree? Why or why not?

(d) Now suppose $\mathcal{C}$ is instead the curve parameterized by $\mathbf{r}(t) = \left\langle \ln t, \sin(\ln t)\sqrt{t^3+1} \right\rangle$ for $1 \leq t \leq e^{2\pi}$. Find $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

3. Find the work done by the force field $\mathbf{F}(x, y) = e^{-y}\mathbf{i} - xe^{-y}\mathbf{j}$ in moving a particle from $(0, 1)$ to $(2, 0)$.

4. Given the vector field $\mathbf{F}(x, y, z) = \langle y^2 \cos z, 2xy \cos z, -xy^2 \sin z \rangle$ and the curve $\mathcal{C}$ parameterized by $\mathbf{r}(t) = \langle t^2, \sin t, t \rangle$ for $0 \leq t \leq 2\pi$, evaluate $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

5. Consider the vector field

$$\mathbf{F}(x,y) = \frac{-y}{x^2+y^2}\mathbf{i} + \frac{x}{x^2+y^2}\mathbf{j}.$$

(a) Parametrizing $\mathcal{C}$, where $\mathcal{C}$ is the unit circle oriented counterclockwise, explicitly calculate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

(b) Show that $\dfrac{\partial F_1}{\partial y} = \dfrac{\partial F_2}{\partial x}$.

(c) Do your answers to parts (a) and (b) contradict each other? Why or why not? *Hint:* If $\mathcal{D}$ is the region in $\mathbb{R}^2$ where $\mathbf{F}$ is defined, is $\mathcal{D}$ simply connected?

6. Consider the vector field

$$\mathbf{F}(x,y,z) = \left\langle \frac{1}{x} + e^{xy}yz, \frac{1}{y} + e^{xy}xz, \frac{1}{z} + e^{xy} + 1 \right\rangle.$$

Show that $\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r} = 0$ for any closed curve $\mathcal{C}$ contained entirely in the first octant.