Math 32B, Fall 2019 **WS - Week 7** **Full Name:**

1. Find the surface area of the part of the surface $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

2. Find the area of the part of the surface $x^2 + y^2 + z^2 = 4z$ that lies inside $z = x^2 + y^2$.

3. Suppose two circular cylinders, each with radius R, intersect at right angles. Consider the solid contained within both cylinders.

   (a) Find the volume of the solid contained within both cylinders in terms of R.

   (b) Find the total surface area of the solid contained within both cylinders in terms of the radius R.

4. Let $\mathcal{D}$ be the domain $\mathcal{D} = \{(u, v) \mid 0 \le u \le 2\pi, -1 \le v \le 1\}$. Consider the parametric surface described by $\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v)\rangle$ where

$$x(u, v) = 2\cos u + v \sin\left(\frac{u}{2}\right)\cos u$$
$$y(u, v) = 2\sin u + v \sin\left(\frac{u}{2}\right)\sin u$$
$$z(u, v) = v \cos\left(\frac{u}{2}\right)$$

Consider the vector $\mathbf{r}(u, v)$ as a sum of two vectors, $\mathbf{c}(u) + \mathbf{s}(u, v)$, where

$$\mathbf{c}(u) = \langle 2\cos u, 2\sin u, 0\rangle \text{ and } \mathbf{s}(u, v) = v\left\langle \sin\left(\frac{u}{2}\right)\cos u, \sin\left(\frac{u}{2}\right)\sin u, \cos\left(\frac{u}{2}\right)\right\rangle.$$

(a) As u varies from 0 to 2π, what curve does $\mathbf{c}(u)$ parameterize in $\mathbb{R}^3$?

(b) What is $||\mathbf{s}(u, v)||$?

(c) What angle does $\mathbf{s}(u, v)$ make with the z-axis? *Hint:* Compute $\mathbf{s} \cdot \mathbf{k}$.

(d) Verify that the projection of $\mathbf{s}(u, v)$ onto the xy-plane is a multiple of $\mathbf{c}(u)$.

(e) For each of the values $v \in \{1, -1\}$ and $u \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$ draw the vector $\mathbf{s}(u, v)$ with tail at $\mathbf{c}(u)$, so that the tip of the vector points to $\mathbf{r}(u, v)$.

(f) Now allow v to vary between -1 and 1, and u to vary between 0 and 2π. What surface is described by $\mathbf{r}(u, v)$?

(g) Compute the normal vector $\mathbf{r}_u \times \mathbf{r}_v$ to the surface for the values $u = 0$ and $u = 2\pi$. Intuitively, this computation shows that the surface is non-orientable.